Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

MAGIC STARS.[1]

A five-pointed star being the smallest that can be made, the rules will be first applied to this one.

Choosing for its constant, or summation $(S)=48$, then:

$$(5\times 48)/2=120=\text{sum of series.}$$

Divide 120 into two parts, say 80 and 40, although many other divisions will work out equally well. Next find a series of five numbers, the sum of which is one of the above two numbers. Selecting 40, the series $6+7+8+9+10=40$ can be used. These numbers must now be written in the central pentagon of the star following the direction of the dotted lines, as shown in Fig. 1.

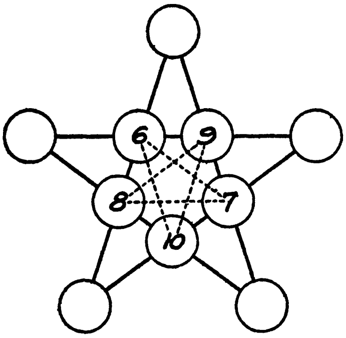


Fig. 1.

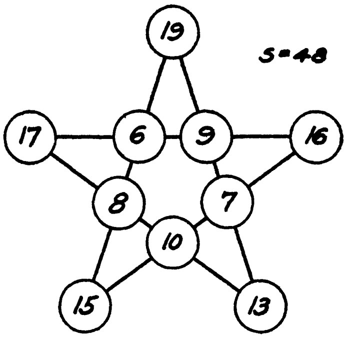


Fig. 2.

Find the sum of every pair of these numbers around the circle beginning in this case with $6+9=15$ and copy the sums in a separate column (A) as shown below:

	(A)
$6+9=15$	$17+15+16=48$
$7+10=17$	$16+17+15=48$
$8+6=14$	$15+14+19=48$
$9+7=16$	$19+16+13=48$
$10+8=18$	$13+18+17=48$

Place on each side of 15, numbers not previously used in the central pentagon, which will make the total of the three numbers $=48$ or S. 17 and 16 are here selected. Copy the last number of the trio (16) under the first number (17) as shown above, and under 16 write the number required to make the sum of the second trio = 48 (in this case 15). Write 15 under 16, and proceed as before to the end. If proper numbers are selected to make the sum of the first trio = 48, it will be found that the first number of the first trio will be the same as the last number of the last trio (in this case 17) and this result will indicate that the star will sum correctly if the numbers in the first column are written in their proper order at the points of the star, as shown in Fig. 2. If the first and last numbers prove different, a simple operation may be used to correct the error. When the last number is *more* than the first number, add half the difference between the two numbers to the first number and proceed as before, but if the last number is *less* than the first number, then *subtract* half the difference from the first number. One or other of these operations will always correct the error.

---

[1] We are indebted to Mr. Frederick A. Morton, Newark, N. J., for these plain and simple rules for constructing magic stars of all orders, and to Mr. H. A. Sayles, Schenectady, N. Y., for drawing the diagrams.

For example, if 14 and 19 had been chosen instead of 17 and 16, the numbers would then run as follows:

$$\begin{aligned}14+15+19&=48\\19+17+12&=48\\12+14+22&=48\\22+16+10&=48\\10+18+20&=48\end{aligned}$$

The difference between the first and last numbers is seen to be 6, and 20 being *more* than 14, half of 6 *added* to 14 makes 17 which is the correct starting number. Again, if 21 and 12 had been selected, then:

$$\begin{aligned}21+15+12&=48\\12+17+19&=48\\19+14+15&=48\\15+16+17&=48\\17+18+13&=48\end{aligned}$$

The difference between the first and last numbers is here 8, and the last number being *less* than the first, half of this difference subtracted from 21 leaves 17 as before.

It is obvious that the constant S of a star of any order may be changed almost indefinitely by adding or subtracting a number selected so as to avoid the introduction of duplicates. Thus, the constant of the star shown in Fig. 2 may be reduced from 48 to

40 by subtracting 4 from each of the five inside numbers, or it may be increased to 56 by adding 4 to each of the five outside numbers, and another variant may then be made by using the five inside numbers of S = 40, and the five outside numbers of S = 56. These three variants are shown respectively in Figs. 3, 4 and 5.

It is also obvious that any pair of five-pointed or other stars may be superposed to form a new star, and by rotating one star over the other, four other variants may be made; but in these and similar operations duplicate numbers will frequently occur, which of course will make the variant ineligible although its constant must necessarily remain correct.

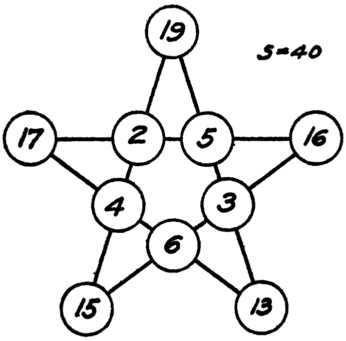


Fig. 3.

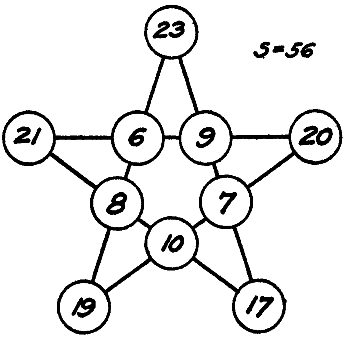


Fig. 4.

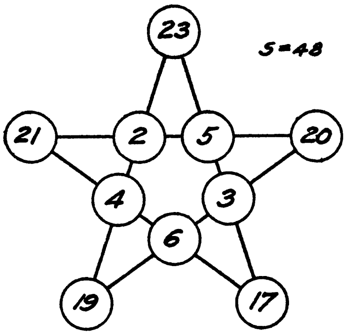


Fig. 5.

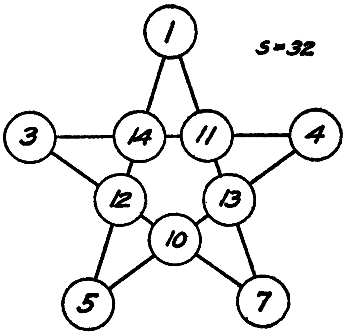


Fig. 6.

Variants may also be made in this and all other orders of magic stars, by changing each number therein to its complement with some other number that is larger than the highest number used in the original star. The highest number in Fig. 2, for example, is 19. Choosing 20 as a number on which to base the desired variant, 19 in Fig. 2 is changed to 1, 17 to 3 and so on throughout, thus making the new-five-pointed star shown in Fig. 6 with S = 32.

The above notes on the construction of variants are given in detail as they apply to *all orders* of magic stars and will not need repetition.

The construction of a six-pointed star may now be considered. Selecting 27 as a constant:

$$(6\times 27)/2 = 81 = \text{sum of the series.}$$

Divide 81 into two parts, say 60 and 21, and let the sum of the six numbers in the inner hexagon = 21, leaving 60 to be divided among the outer points. Select a series of six numbers, the sum of which is 21, say 1, 2, 3, 4, 5, 6, and arrange these six numbers in hexagonal form, so that the sum of each pair of opposite numbers = 7. Fig. 7 shows that these six inside numbers form part of two triangles, made respectively with single and double lines. The outside numbers of each of these two triangles must be computed separately according to the method used in connection with the five-pointed star. Beginning with the two upper numbers in the single-lined triangle and adding the couplets together we have:

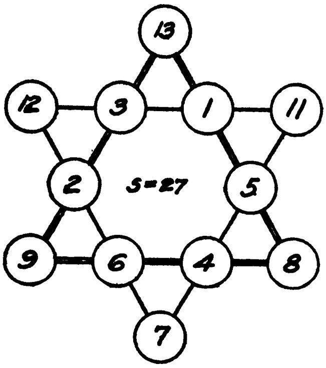


Fig. 7.

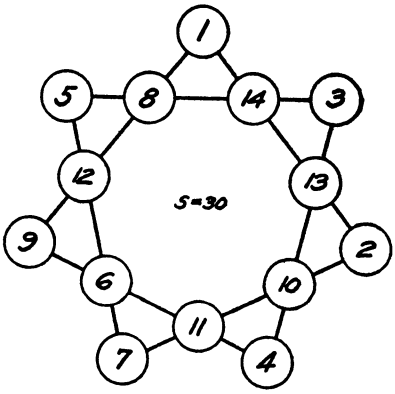


Fig. 8.

(A)

$3+1=4$ $\qquad$ $12+4+11=27$

$5+4=9$ $\qquad$ $11+9+7=27$

$6+2=8$ $\qquad$ $7+8+12=27$

Writing these sums in a separate column (A) and proceeding as before described, the numbers 12, 11, 7 are obtained for the points of the single-lined triangle, and in the same manner 13, 8, 9 are found for the points of the double-lined triangle, thus completing the six-pointed star Fig. 7.

The next larger star has seven points. Selecting 30 for a constant, which is the lowest possible:

$$(7\times 30)/2=105=\text{sum of the series.}$$

Dividing this sum as before into two parts, say 31 and 74, seven numbers are found to sum 74, say, $6+8+10+11+12+13+14=74$, and these numbers are written around the inside heptagon as shown in Fig. 8. Adding them together in pairs, their sums are written in a column and treated as shown below, thus determining the numbers for the points of Fig. 8.

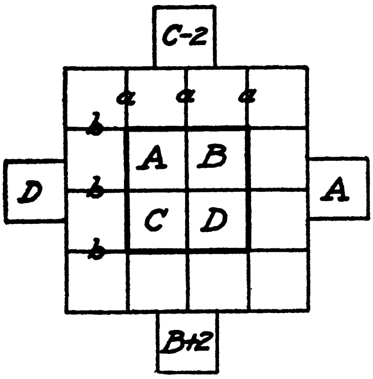


Fig. 9.

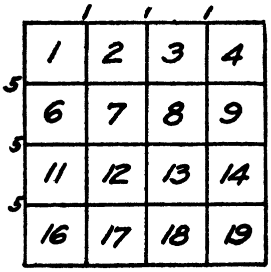


Fig. 10.

$14+13=27$	$1+27+2=30$
$10+11=21$	$2+21+7=30$
$6+12=18$	$7+18+5=30$
$8+14=22$	$5+22+3=30$
$13+10=23$	$3+23+4=30$
$11+6=17$	$4+17+9=30$
$12+8=20$	$9+20+1=30$

The next larger star has eight points and it can be made in two different ways, viz., By arranging the numbers in one continuous line throughout as in stars already described having an odd number of points, or by making it of two interlocking squares.

The latter form of this star may be constructed by first making a 4^2 with one extra cell on each of its four sides, as shown in Fig. 9. A series of sixteen numbers is then selected which will meet the conditions shown by italics *a, a, a,* and *b, b, b,* in the figure, i. e., all differences between row numbers must be the same, and also all differences between column numbers, but the two differences must be unlike. The constant (S) of the series when the latter is arranged as a magic 4^2 must also be some multiple of 4. The series is then put into magic formation by the old and well-known rule for making magic squares of the 4th order. The central 2×2

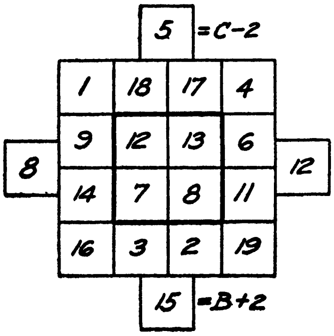


Fig. 11.

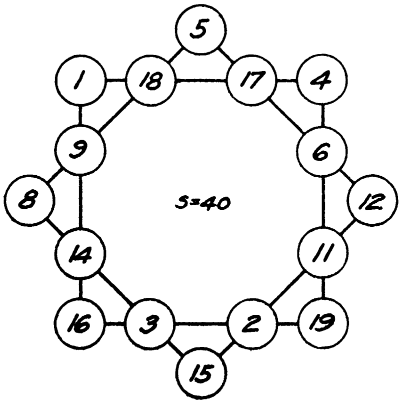


Fig. 12.

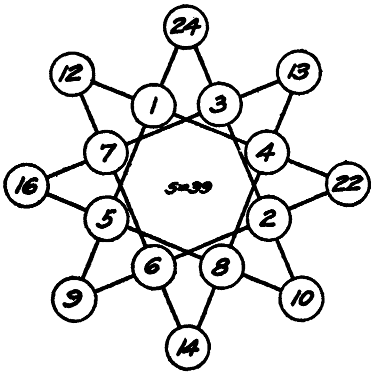


Fig. 13.

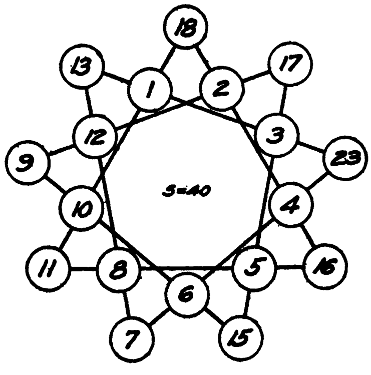


Fig. 14.

square is now eliminated and the numbers therein transferred to the four extra outside cells as indicated by the letters A. B. C. D. Finally all numbers are transferred in their order into an eight-pointed star.

A series of numbers meeting the required conditions is shown in Fig 10, and its arrangement according to the above rules is given in Fig. 11, the numbers in which, transferred to an eight-pointed star, being shown in Fig. 12, S=40. The 4^2 magic arrangement of the series must be made in accordance with Fig. 11, for other magic arrangements will often fail to work out, and will never do so in accordance with Fig. 9. The above instructions cover the simplest method of making this form of star but it can be constructed in

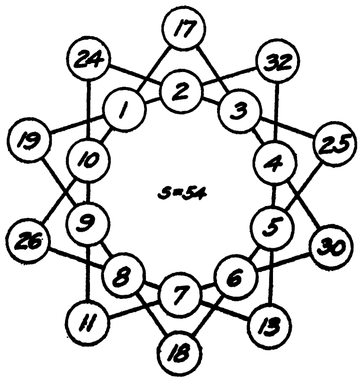


Fig. 15.

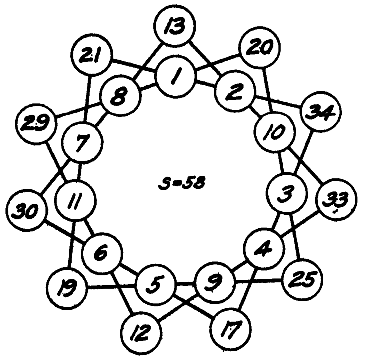


Fig. 16.

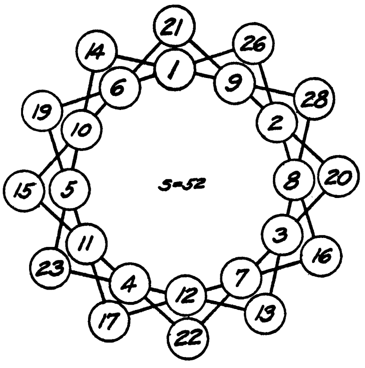


Fig. 17.

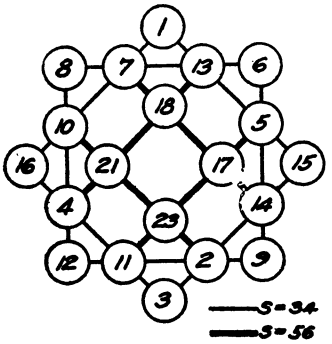
many other different ways and also with constants which are not evenly divisible by 4.

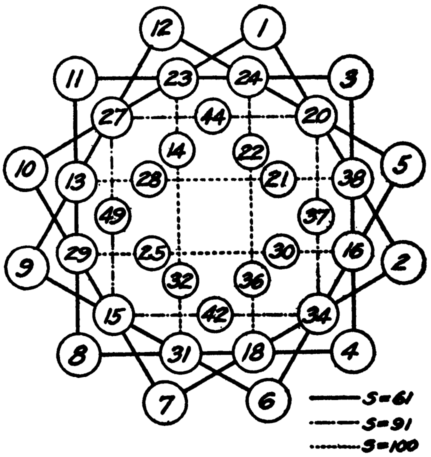
Turning now to the construction of the eight-pointed star by the continuous line method, inspection of Figs. 12 and 13 will show that although the number of points is the same in each star yet the arrangement of numbers in their relation to one another in the eight quartets is entirely different.

Choosing a constant of 39 for an example:

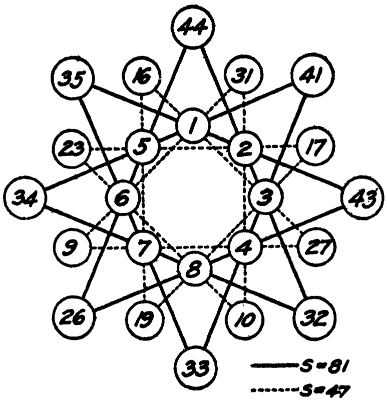
$$(39\times 9)/2 = 156 = \text{sum of series.}$$

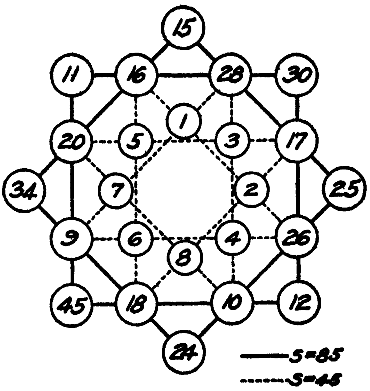
This sum is now divided into two parts, say 36 and 120. The sum of the first eight digits being 36, they may be placed around the inside octagon so that the sum of each opposite pair of numbers = 9, as shown in Fig. 13. Adding them together in pairs, as indicated by the connecting lines in the figure, their sums are written in a column and treated as before explained, thus giving the correct numbers to be arranged around the points of the star Fig. 13.

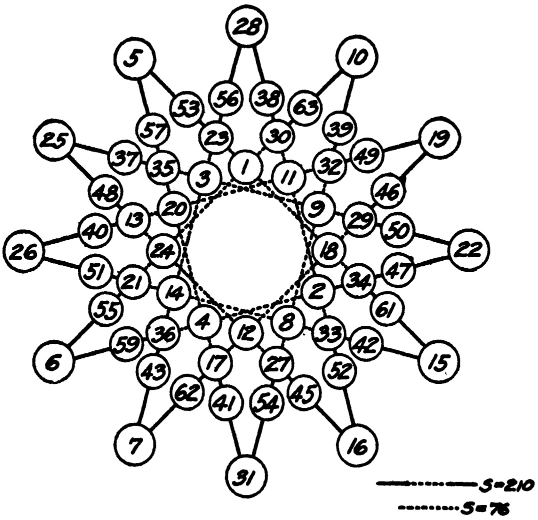


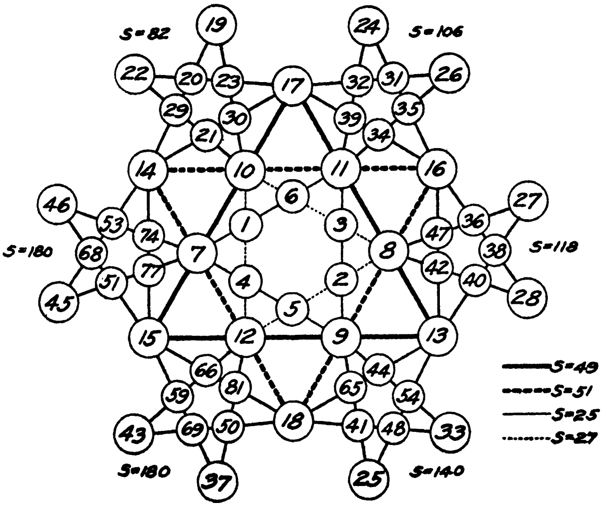


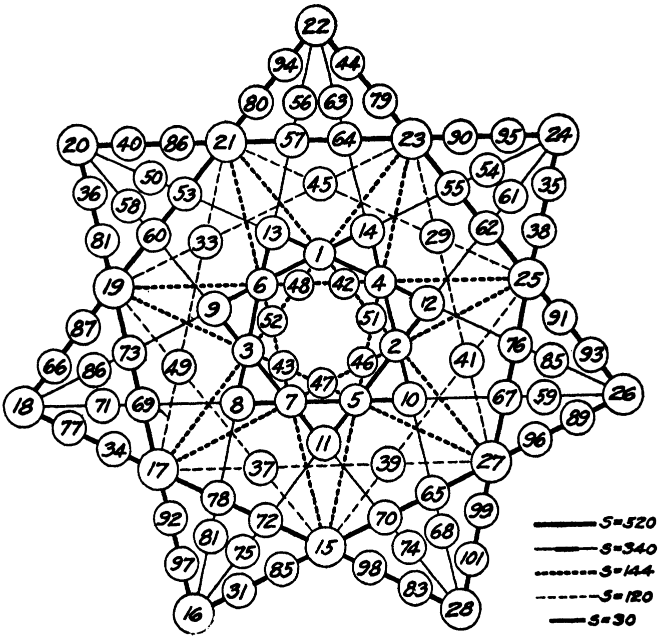
These rules for making magic stars of all orders are so simple that further examples are deemed unnecessary. Nine-, ten-, eleven-, and twelve-pointed stars, made by the methods described, are shown respectively in Figs. 14, 15, 16, and 17. Several other diagrams of ingenious and more intricate star patterns made by Mr. Morton are also appended for the interest of the reader.

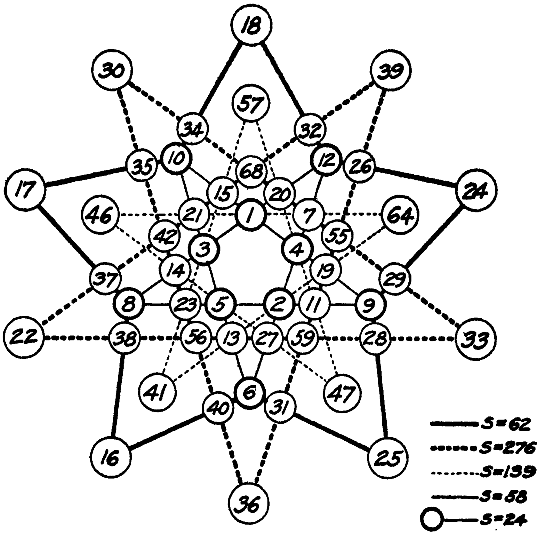

S=81
S=47

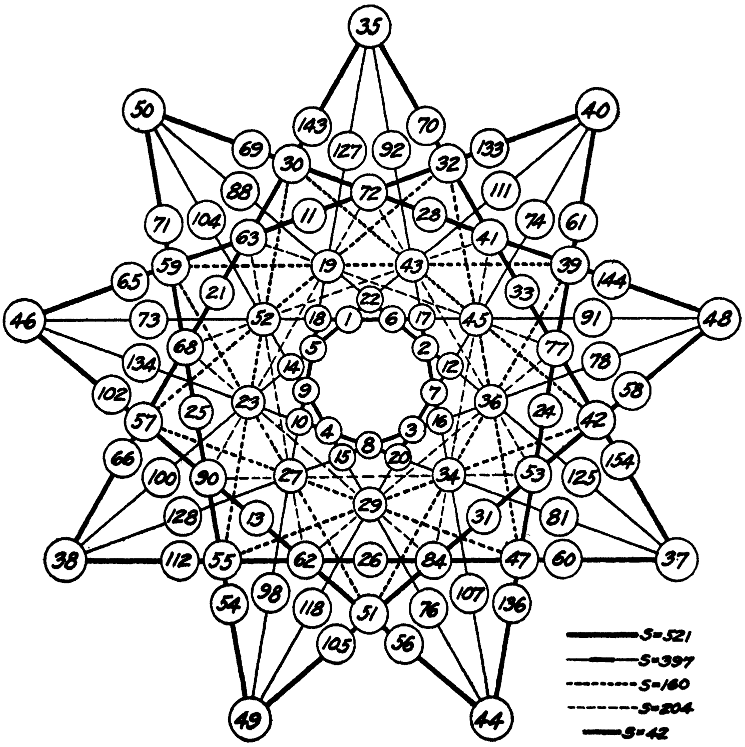

S=85
S=45

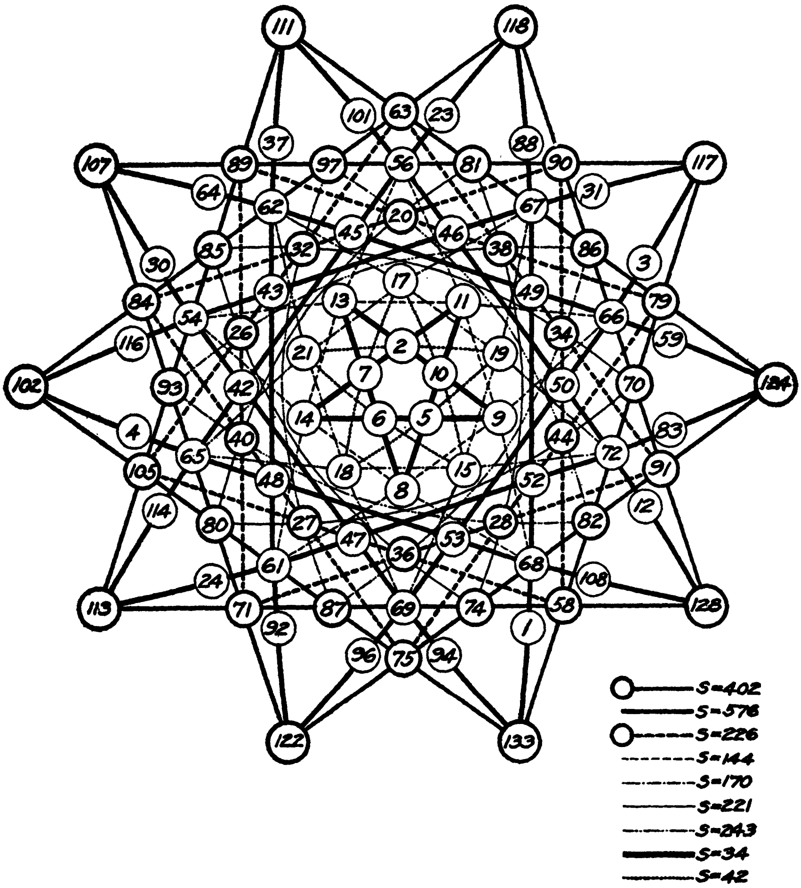

S=210
S=76


S=82
S=106
S=180
S=118
S=180
S=140
S=49
S=51
S=25
S=27


S=520
S=340
S=144
S=120
S=30


S=62
S=276
S=139
S=58
S=24


S=521
S=397
S=160
S=204
S=42



W. S. ANDREWS.

SCHENECTADY, N. Y.

RECENT PERIODICALS.

In *Scientia* (*Rivista di Scienza*) for May, 1914, the first artilcle is by A. Einstein on the principle of relativity. It is only fair, after the criticisms of Marcel Brillouin and Max Abraham, that an article written by one of the chief advocates of the theory should appear in *Scientia.* Einstein distinguishes two theories of relativity: the theory in the "narrow sense" and that in the "wide sense." If we refer a motion to a system of coordinates K, for which the Newtonian equations are valid, any other system of